

Masking singularities with k -essence fields in an emergent gravity metric

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It is known that dynamical solutions of the k -essence equation of motion change the metric for the perturbations around these solutions and the perturbations propagate in an emergent spacetime with metric $\tilde{G}^{\mu\nu}$ different from the gravitational metric $g^{\mu\nu}$. We show that for observers travelling with the perturbations, there exist homogeneous field configurations for the lagrangian $L = [\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi]^2$ for which a singularity in the gravitational metric $g^{\mu\nu}$ can be masked or hidden for such observers. This is shown for the Schwarzschild and the Reissner-Nordstrom metrics.

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1. Introduction

Present day observations have established that the universe consists of roughly 25 percent dark matter, 70 percent dark energy, about 4 percent free hydrogen and helium with the remaining one percent consisting of stars, dust, neutrinos and heavy elements. Actions with non-canonical kinetic terms have been shown to be strong candidates for dark matter and dark energy. A theory with a non-canonical kinetic term was first proposed by Born and Infeld in order to get rid of the infinite self-energy of the electron [1]. Similar theories were also studied in [2, 3]. Cosmology witnessed these models first in the context of scalar fields having non-canonical kinetic terms which drive inflation. Subsequently k -essence models of dark matter and dark energy were also constructed [4–11]. Effective field theories arising from string theories also have non-canonical kinetic terms [12–15].

An approach to understand the origins of dark matter and dark energy involve setting up lagrangians for what are known as k -essence fields in a Friedman-Robertson-Walker metric with zero curvature constant. In one approach [16] it is possible to unify the dark matter and dark energy components into a single scalar field model with the scalar field ϕ having a non-canonical kinetic term. These scalar fields are the k -essence fields mentioned above. The general form of the lagrangian for these k -essence models is assumed to be a function $F(X)$ with $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, and do not depend explicitly on ϕ to start with. In [16], X was shown to satisfy a general scaling relation, *viz.* $X(\frac{dF}{dX})^2 = Ca(t)^{-6}$ with C a constant (similar expression was also derived in [17]).

Recently a lagrangian for the k -essence field has been set up [18] in a homogeneous and isotropic universe where there are two generalised coordinates $q(t) = \ln a(t)$ ($a(t)$ is the scale factor) and a scalar field $\phi(t)$ with a complicated polynomial interaction between them. In the lagrangian, q has a standard kinetic term while ϕ does

not have a kinetic part and occurs purely through the interaction term. [18] incorporates the scaling relation of [16]. In [19] questions regarding the amplitude of a scale factor at some epoch evolving to a different value at a later epoch was addressed for the above lagrangian at times close to the big bang (very small scale factor). As the scale factor is inversely proportional to the temperature at a particular epoch, these amplitudes provided an estimate of quantum fluctuations of the temperature.

Relativistic field theories with canonical kinetic terms differ from lagrangian theories of k -essence in that non-trivial dynamical solutions of the k -essence equation of motion not only spontaneously break Lorentz invariance but also change the metric for the perturbations around these solutions [20]. The perturbations propagate [20, 21] in an emergent spacetime with metric $\tilde{G}_{\mu\nu}$ different from (and also not conformally equivalent to) the gravitational metric $g_{\mu\nu}$.

Now $g_{\mu\nu}$ can contain physical singularities. The motivation of this work is to investigate whether scenarios can be constructed where the singularity in $g_{\mu\nu}$ can be "masked" to observers travelling piggy-back on the perturbations of the k -essence scalar fields. Lagrangians for k -essence scalar fields have the general form [20], $L(X, \phi) = V(\phi) + K_1(\phi)X + K_2(\phi)X^2 + \dots$ In [20] various scenarios have been described for this lagrangian including those linear in X . We show here that a simple model lagrangian quadratic in X , *viz.* $L = X^2$ has k -essence field configurations (for both the Schwarzschild and the Reissner-Nordstrom metrics) for which the singularities can be masked for observers sitting on the scalar field perturbations. The plan of the paper is as follows. In Section 2 a brief summary is given of emergent gravity concepts as developed in [20]. In Section 3 the Schwarzschild metric is considered while Section 4 deals with the Reissner-Nordstrom case. Section 5 is the conclusion.

EMERGENT GRAVITY

Consider the k -essence scalar field ϕ minimally coupled to the gravitational field $g_{\mu\nu}$. Then the k -essence action is +

$$S_k[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} L(X, \phi) \quad (1)$$

where $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ and ∇_μ means the covariant derivative associated with the metric $g_{\mu\nu}$. The total action describing the dynamics of k -essence and gravity is

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[-\frac{1}{2}M_{\text{Pl}}^2 R + L(X, \phi) \right] \quad (2)$$

where R is the Ricci scalar and M_{Pl} the reduced Planck mass. The energy momentum tensor for the k -essence field is (with $L_X = \frac{dL}{dX}$, $L_{XX} = \frac{d^2L}{dX^2}$, $L_\phi = \frac{dL}{d\phi}$)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_k}{\delta g^{\mu\nu}} = L_X \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}L \quad (3)$$

and the equation of motion for the k -essence field is

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \tilde{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi + 2X L_{X\phi} - L_\phi = 0 \quad (4a)$$

where the effective metric $\tilde{G}^{\mu\nu}$ is

$$\tilde{G}^{\mu\nu} = L_X g^{\mu\nu} + L_{XX} \nabla^\mu\phi \nabla^\nu\phi \quad (5a)$$

and is physically meaningful only when

$$1 + \frac{2X L_{XX}}{L_X} > 0$$

i.e the sound speed $c_s = (1 + \frac{2X L_{XX}}{L_X})^{-1/2}$ is a real quantity. When this condition holds everywhere the effective metric $\tilde{G}^{\mu\nu}$ determines the characteristics for k -essence [8, 22–24]. For the non-trivial configurations of the k -essence field $\partial_\mu\phi \neq 0$ and $\tilde{G}^{\mu\nu}$ is not conformally equivalent to $g^{\mu\nu}$. So the characteristics are different from canonical scalar fields whose lagrangians are linear in X . The characteristics determine the local causal structure of the spacetime at every point of the manifold. So the local causal structure for the k -essence field is different from those ones defined by $g^{\mu\nu}$.

Making a conformal transformation $G^{\mu\nu} \equiv \frac{c_s}{L_X} \tilde{G}^{\mu\nu}$ and using the expression for $T_{\mu\nu}$ from equation (3) one can write the inverse of the metric $G^{\mu\nu}$ as

$$G_{\mu\nu} = \frac{L_X}{c_s} g_{\mu\nu} - c_s L_{XX} \nabla_\mu\phi \nabla_\nu\phi \quad (5b)$$

We will be using this expression for the effective metric in all that follows. Also note that after this conformal transformation, if we further assume that L is not an explicit function of ϕ then the equation of motion (4a) is replaced by ;

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \frac{L_X^2}{c_s} G^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0 \quad (4b)$$

THE SCHWARZSCHILD SOLUTION

The Schwarzschild metric is given by ($r_s = 2GM/c^2 \equiv 2GM$, taking $c = 1$)

$$ds^2 = (1 - \frac{r_s}{r}) dt^2 - (1 - \frac{r_s}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\Phi^2) \quad (6)$$

and the emergent metric components $G_{\mu\nu}$ are related to the Schwarzschild metric components $g_{\mu\nu}$ by (5b). Therefore for $L = X^2$, and assuming the k -essence field to be spherically symmetric i.e. $\phi \equiv \phi(r, t)$ one has

$$G_{00} = (1 - \frac{r_s}{r}) 2\sqrt{3}X - \frac{2}{\sqrt{3}} (\frac{\partial\phi}{\partial t})^2$$

$$G_{11} = -(1 - \frac{r_s}{r})^{-1} 2\sqrt{3}X - \frac{2}{\sqrt{3}} (\frac{\partial\phi}{\partial r})^2$$

$$G_{22} = 2\sqrt{3}X g_{22} = -2\sqrt{3}X r^2$$

$$G_{33} = 2\sqrt{3}X g_{33} = -2\sqrt{3}X r^2 \sin^2\theta \quad (7)$$

$$G_{01} = G_{10} = -\frac{2}{\sqrt{3}} \frac{\partial\phi}{\partial t} \frac{\partial\phi}{\partial r} \quad (8)$$

All the other $G_{\mu\nu}$ are zero.

Assume $\phi(r, t)$ to be of the form $\phi_s(r, t) = \phi_{1s}(r) + \phi_{2s}(t)$. Then

$$G_{00} = (1 - \frac{r_s}{r}) 2\sqrt{3}X - \frac{2}{\sqrt{3}} (\frac{d\phi_{2s}}{dt})^2$$

$$G_{11} = -(1 - \frac{r_s}{r})^{-1} 2\sqrt{3}X - \frac{2}{\sqrt{3}} (\frac{d\phi_{1s}}{dr})^2 \quad (9)$$

$$G_{01} = G_{10} = -\frac{2}{\sqrt{3}} \dot{\phi}_{2s} \phi'_{1s} \quad (10)$$

where "dot" denotes differentiation with respect to time and the "prime" is differentiation with respect to r . As we are concerned only with the singularity structure of the metrics we are not discussing the G_{22} and G_{33} components as g_{22} and g_{33} are well behaved for $r \rightarrow 0$.

We assume that L_X ; L_{XX} ; $L_{XX}(\frac{\partial\phi}{\partial t})^2$; $L_{XX}(\frac{\partial\phi}{\partial r})^2$ are all well behaved quantities for $r \rightarrow 0$. All these conditions hold true in the above equations if we also assume that $(\frac{\partial\phi_{1s}}{\partial r})$ is well behaved for $r \rightarrow 0$. We shall consider only physical singularities. Here these occur at $r = 0$. The singularities at $r = r_s$ are coordinate singularities and these can always be removed by some coordinate transformations and we are not considering them.

Note that at $r = 0$, the second terms on the *r.h.s.* of (9) and (10) are well behaved as per our assumptions. Therefore good behaviour of G_{00} and G_{11} at $r = 0$ is guaranteed if there exist two functions $f_1(r), f_2(r)$ such that both these functions are well behaved at $r = 0$ and

$$(1 - \frac{r_s}{r})2\sqrt{3}X = f_1(r) ; (1 - \frac{r_s}{r})^{-1}2\sqrt{3}X = f_2(r) \quad (11)$$

These equations imply that $f_1(r) = f_2(r)(1 - \frac{r_s}{r})^2$. It is readily seen that for X to be well behaved as $r \rightarrow 0$ and for the two equations in (11) to be consistent one possibility is $f_1(r) = \text{constant} = 1$ and $f_2(r) = (1 - \frac{r_s}{r})^{-2}$. Then X is well behaved at $r = 0$. So

$$G_{00} = 1 - \frac{2}{\sqrt{3}}(\dot{\phi}_{2s})^2 ; G_{11} = -(1 - \frac{r_s}{r})^{-2} - \frac{2}{\sqrt{3}}(\phi'_{1s})^2 \quad (12)$$

At $r = 0$ both G_{00} , G_{11} are well behaved and

$$X = \frac{1}{2\sqrt{3}} \frac{1}{(1 - \frac{r_s}{r})} \quad (13)$$

With our assumption regarding the form of $\phi(r, t)$, this leads to

$$(\dot{\phi}_{2s}(t))^2 = \frac{1}{\sqrt{3}} + (1 - \frac{r_s}{r})^2(\phi'_{1s}(r))^2 = k \quad (14)$$

where k is a constant. Note that (12) and (14) imply that if the sign of (temporal component) G_{00} has to remain positive w.r.t. (spatial components) G_{11}, G_{22}, G_{33} , then $G_{00} > 0$. This means $k < \frac{3}{4} = 0.75$. Only these values of k are allowed.

We now discuss possible solutions to this equation. Note that

Case 1, $k = 0$

We rule out taking $k = 0$ because then $\dot{\phi}_{2s}(t) = 0$ which means that the k -essence scalar field does not have any kinetic energy. This violates the basic premise of k -essence where the kinetic energy drives the accelerated expansion.

Case 2, $k = \frac{1}{\sqrt{3}} = 0.5773 < 0.75$

Now we have , $\phi'_{1s} = 0$ and $\dot{\phi}_{2s} = (3)^{-1/4}$ so that

$$\phi_{1s}(r) = c_1 ; \phi_{2s}(t) = (3)^{-1/4}t + c_2 \quad (15)$$

where c_1, c_2 are constants. Now $X = \frac{1}{2\sqrt{3}} \frac{1}{(1 - \frac{r_s}{r})}$ and $G_{00} = 1 - \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} = \frac{1}{3} \neq g_{00}$; $G_{11} = -\frac{1}{(1 - \frac{r_s}{r})^2} \neq g_{11}$; $G_{22} = 2\sqrt{3}Xg_{22}$; $G_{33} = 2\sqrt{3}Xg_{33}$; $G_{01} = G_{10} = 0$. All the other off-diagonal components are also zero. So the emergent metric without any singularity at $r = 0$ is

$$G_{\mu\nu} = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{-1}{(1 - \frac{r_s}{r})^2} & 0 & 0 \\ 0 & 0 & \frac{-r^2}{(1 - \frac{r_s}{r})} & 0 \\ 0 & 0 & 0 & \frac{-r^2 \sin^2 \theta}{(1 - \frac{r_s}{r})} \end{pmatrix} \quad (16)$$

It is straightforward to see from eqs.(16) that $G_{\mu\nu}$ and $g_{\mu\nu}$ are not conformally equivalent.

Therefore there exist homogeneous (i.e. independent of r) k -essence scalar field configurations, *viz.* $\phi(r, t) = c_1 + (3)^{-1/4}t + c_2$, that can give rise to an emergent gravity metric where the singularity in the gravitational metric $g_{\mu\nu}$ is masked for observers riding on the scalar field perturbations. These configurations also satisfy the emergent gravity equations of motion (4b) as is easily seen: $\frac{L^2}{c_s} [G^{00}\partial_0^2\phi_{2s} + G^{11}(\partial_1^2\phi_{1s} - \Gamma_{11}^1\partial_1\phi_{1s}) + G^{01}\nabla_0\nabla_1\phi + G^{10}\nabla_1\nabla_0\phi] = 0$. The first two terms within third brackets vanish because ϕ_{2s} is linear in t and ϕ_{1s} is a constant. The last two terms vanish because $G^{01}\nabla_0\nabla_1\phi + G^{10}\nabla_1\nabla_0\phi = G_{01}\nabla^0\nabla^1\phi + G_{10}\nabla^1\nabla^0\phi$ and $G_{01} = G_{10} = 0$.

THE REISSNER-NORDSTROM BLACK HOLE

For a static charged black hole with charge Q the metric is the Reissner-Nordstrom metric:

$$ds^2 = (1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})dt^2 - (1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\Phi^2) \quad (17)$$

with $r_Q^2 = GQ^2/4\pi\epsilon_0 c^4 \equiv GQ^2/4\pi\epsilon_0$ taking $c = 1$. We now carry out an exactly similar analysis as before for the same lagrangian $L = X^2$ and assume the solutions for ϕ to be of the form $\phi_n(r, t) = \phi_{1n}(r) + \phi_{2n}(t)$. Then

$$G_{00} = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)2\sqrt{3}X - \frac{2}{\sqrt{3}}\left(\frac{d\phi_{2n}}{dt}\right)^2$$

$$G_{11} = -\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1}2\sqrt{3}X - \frac{2}{\sqrt{3}}\left(\frac{d\phi_{1n}}{dr}\right)^2 \quad (18a)$$

$$G_{01} = G_{10} = -\frac{2}{\sqrt{3}}\dot{\phi}_{2n}\phi'_{1n} ; G_{22} = 2\sqrt{3}Xg_{22} ; G_{33} = 2\sqrt{3}Xg_{33} \quad (18b)$$

All the other $G_{\mu\nu}$ are zero.

As before at $r = 0$, the second terms on the *r.h.s.* of (18a) are well behaved as per our assumptions. So good behaviour of G_{00} and G_{11} at $r = 0$ is guaranteed if there exist two functions $g_1(r), g_2(r)$ such that both these functions are well behaved at $r = 0$ and

$$(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})2\sqrt{3}X = g_1(r) ; (1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^{-1}2\sqrt{3}X = g_2(r) \quad (19)$$

These equations imply that $g_1(r) = g_2(r)(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^2$. For X to be well behaved as $r \rightarrow 0$ and for consistency one possibility is $g_1(r) = \text{constant} = 1$ and $g_2(r) = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-2}$.

$\frac{r_s}{r} + \frac{r_Q^2}{r^2} \Big)^{-2}$. Then X is well behaved at $r = 0$. So

$$G_{00} = 1 - \frac{2}{\sqrt{3}}(\dot{\phi}_{2n})^2 ; G_{11} = -(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^{-2} - \frac{2}{\sqrt{3}}(\phi'_{1n})^2 \quad (20)$$

At $r = 0$ both \tilde{G}_{00} , \tilde{G}_{11} are well behaved and

$$X = \frac{1}{2\sqrt{3}} \frac{1}{(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})} \quad (21)$$

With our assumption regarding the form of $\phi(r, t)$, this leads to

$$(\dot{\phi}_{2n}(t))^2 = \frac{1}{\sqrt{3}} + (1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^2 (\phi'_{1n}(r))^2 = k \quad (22)$$

where k is a constant. For $k = \frac{1}{\sqrt{3}}$, again $\phi'_{1n}(r) = 0$ and we can write the solution for the k -essence field as a homogeneous field $\phi(r, t) = d_1 + (3)^{-1/4}t + d_2$, where $d_{1,2}$ are constants.

These configurations again satisfy the emergent gravity equations of motion (4b) as before: $\frac{L^2}{c_s^2}[G^{00}\partial_0^2\phi_{2n} + G^{11}(\partial_1^2\phi_{1n} - \Gamma_{11}^1\partial_1\phi_{1n}) + G^{01}\nabla_0\nabla_1\phi + G^{10}\nabla_1\nabla_0\phi] = 0$. The first two terms within third brackets vanish because ϕ_{2n} is linear in t and ϕ_{1n} is a constant. The last two terms vanish because $G^{01}\nabla_0\nabla_1\phi + G^{10}\nabla_1\nabla_0\phi = G_{01}\nabla^0\nabla^1\phi + G_{10}\nabla^1\nabla^0\phi$ and $G_{01} = G_{10} = 0$.

The emergent metric in the case of Reisner-Nordstrom background is then :

$$G_{\mu\nu} = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{-1}{(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})^2} & 0 & 0 \\ 0 & 0 & \frac{-r^2}{(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})} & 0 \\ 0 & 0 & 0 & \frac{-r^2 \sin^2 \theta}{(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2})} \end{pmatrix} \quad (23)$$

"MASKING"

We now briefly discuss the "masking" of the singularity (we ignore the singularity at $r = r_s$ which is a coordinate singularity as already mentioned in the beginning). In our treatment, the scalar field is a homogeneous scalar field , i.e., depends only on the time coordinate. As the spatial part of the scalar field is always a constant in r and the temporal part is linear in the temporal coordinate, our scalar field is well behaved at the central (physical) singularity, *viz.*, $r = 0$.

The perturbations of the scalar field travels in $G_{\mu\nu}$. This metric $G_{\mu\nu}$ is perfectly well behaved at $r = 0$ as can be easily seen. Hence in this metric one can never "see" the physical singularity. This is what we mean by "masking" of the singularity at $r = 0$.

Does this mean that we have done away with the singularity. The answer is obviously *no* as we now show. For illustrative purposes we shall confine ourselves to the Schwarzschild case. Let us define $\delta g_{\mu\nu} = G_{\mu\nu} - g_{\mu\nu}$. Then it is easy to see that

$$\delta g_{00} = \frac{r_s}{r} - \frac{2}{3} ; \delta g_{11} = \frac{-r_s r}{(r - r_s)^2}$$

$$\delta g_{22} = \frac{-r_s r^2}{(r - r_s)} ; \delta g_{33} = \frac{-r_s r^2 \sin^2 \theta}{(r - r_s)} \quad (24)$$

For the Reisner-Nordstrom background the above equations take the form:

$$\delta g_{00} = \frac{r_s}{r} - \frac{r_Q^2}{r^2} - \frac{2}{3} ; \delta g_{11} = \frac{-r_s r^3 + r_Q^2 r^2}{(r^2 - r r_s + r_Q^2)^2}$$

$$\delta g_{22} = \frac{-r_s r^3 + r_Q^2 r^2}{(r^2 - r r_s + r_Q^2)} ; \delta g_{33} = \frac{(-r_s r^3 + r_Q^2 r^2) \sin^2 \theta}{(r^2 - r r_s + r_Q^2)} \quad (25)$$

Note that in both the above examples the change in the original metric $\delta g_{\mu\nu}$ still carries the same singularity structure at $r = 0$ as $g_{\mu\nu}$. This is as it should be. Therefore the singularity is still there but it is impossible to be aware of it if we use $G_{\mu\nu}$. This is what we call "masking".

CONCLUSION

In this work we have shown that for observers whose world line is in an emergent gravity metric $G_{\mu\nu}$,

(a) The physical singularity at $r = 0$ in the gravitational metric $g_{\mu\nu}$ can remain masked for certain configurations of the k -essence field ϕ and observers travelling with the perturbations of such k -essence fields will never be aware of the physical singularity of the gravitational metric as this is not conformally equivalent to the emergent gravity metric.

(b) These configurations are homogeneous (i.e. functions of time t only) and satisfy the equations of motion in the emergent gravity metric.

(c) The above have been shown here for the Schwarzschild and the Reissner-Nordstrom metrics.

Back reaction effects and inhomogeneous field configurations will be discussed in future communications.

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